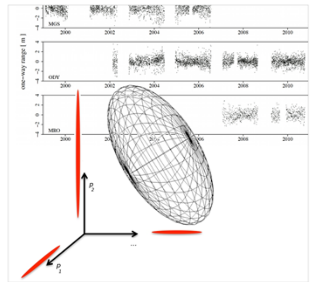
Determining asteroid masses from planetary range

a short course in parameter estimation



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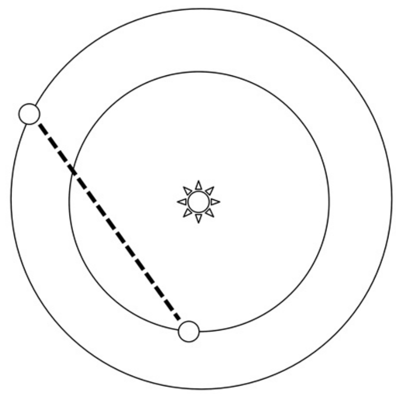
I wish to provide a fresh look on a old problem : how to determine asteroid masses from planetary range observations ?

I also wish to provide a fresh look on parameter estimation altogether -> the presentation should be interesting even for those of you not directly involved in estimating asteroid masses.

- why do we want to determine asteroid masses ?
  - precious source of information about the asteroids themselves
    determination of porosity, material composition, collisional evolution etc.
  - limiting factor in the prediction capacity of planetary ephemerides

- what is ranging data ?
  - ranging data
    measurements of the distance between a spacecraft and a DSN antenna,
    in practice round-trip light-time
  - planetary ranging data
    measurements of the distance between a planet and a DSN antenna



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* ranging data are measurements of the distance between a ref point on an antenna on Earth and a ref point on a spacecraft

based on a signal emitted and bounced back -> from the round-trip time we can compute the distance

* planetary range = ... = range between planetary barycenters

Mars ranging : more than 10 years of data with accuracy ~ 1 m

data with lower accuracy is available since the 1980s

Mars Global Surveyor (MGS)
1999 - 2007

Mars Odyssey (MO)
2002 - now

Mars Reconnaissance Orbiter (MRO)
2006 - now

up to about 400 asteroids could induce perturbations > 2 m

OBJECTIVE : extract asteroid mass information from Mars range data

general introduction to parameter estimation, practical application to asteroid mass determination

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Asteroids are in the main-belt, between Mars and the Earth.

Mars ranging most sensitive to asteroid perturbations

data available with an accuracy of 2 m ...

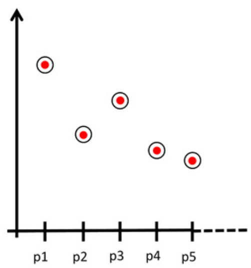
Up to 400 asteroids can perturb ->

how to extract information from the ranging data ?.. the naive approach

model able to reproduce the data

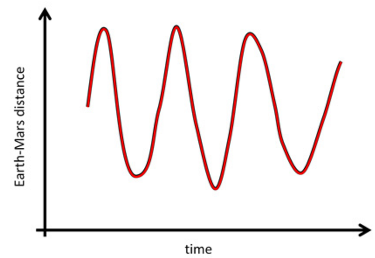
- orbits of the Earth and Mars around the Sun
- perturbers : Moon, planets, asteroids
- other (solar oblateness, solar plasma etc.)

find the parameters of the model, so that range prediction = range data



○ = real value of a parameter (unknown)

● = value of a parameter in the model



the parameters can be recovered by simply exploring the parameter space

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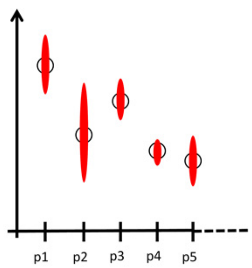
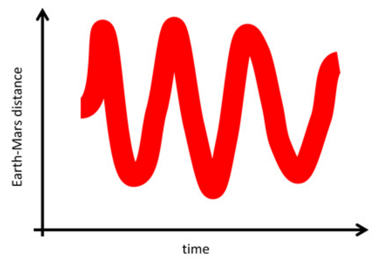
- if data is noisy, many model predictions are satisfactory





➡ uncertainty in observations induces uncertainty in the recovery process.

- how to extract information from the ranging data ?.. a more formal approach

  - notation :

z	range prediction
$P = (p_1, p_2, ...)$	model parameters
z_{obs}	range data

  - if each model parameter is a correction with respect to a reference value, the range dependence on the model parameters is linear :

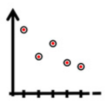
$$z(p_1, p_2, ...) = M_1 p_1 + M_2 p_2 + ... = MP$$

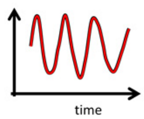
$M_1, M_2, ...$	partials with respect to the parameters
M	matrix of partials

- the distance between data and prediction can be estimated with the L_2 norm

$$\left|z - z_{obs}\right|_2 = \sum_i (z_i - z_{obs,i})^2$$

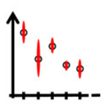
- without noise :

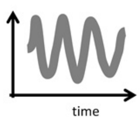




$$\left|z - z_{obs}\right|_2 = (MP - z_{obs})^2 = 0$$
$$\dots$$
$$\underbrace{P = (M^T M)^{-1} M^T z_{obs}}_{\text{least – square solution}}$$

- with noise :





$$\left|z - z_{obs}\right|_2 = (MP - z_{obs})^2 < \varepsilon$$
$$\dots$$
$$(P - P^*)^T M^T M (P - P^*) < \varepsilon^*$$
$$(M^T M)^{-1} M^T z_{obs}$$

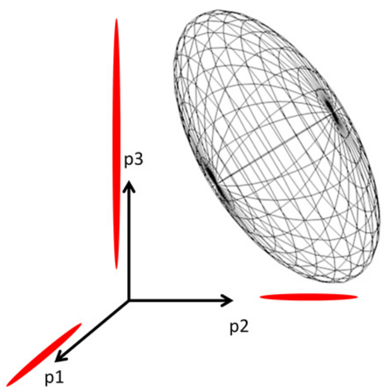
constraint on parameters : $(P-P^*)^T M^T M(P-P^*) < \varepsilon^*$

... interior of an ellipsoid !

if $M^T M$ is diagonal :

$$\left(\frac{p_1-p_1^*}{\sigma_1^{-1}}\right)^2 + \left(\frac{p_2-p_2^*}{\sigma_2^{-1}}\right)^2 + \left(\frac{p_3-p_3^*}{\sigma_3^{-1}}\right)^2 < \varepsilon^*$$

if not diagonal, the ellipsoid is not aligned with the parameter axes



center defined by least-square solution $P^* = (M^T M)^{-1} M^T z_{obs}$

size given by noise ε^*

relative sizes of axes given by proper values of $M^T M$

orientation given by proper vectors of $M^T M$

parameter uncertainties = $\varepsilon^* \sqrt{diag\ (M^T M)^{-1}}$

corresponds to the sides of a bounding box

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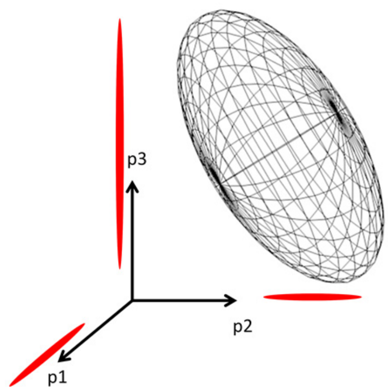
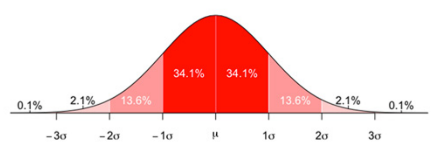
If noise on data follows a Gaussian distribution with variance σ_{ε}^{2}

the probability of finding the real parameters at a given point of the parameter space follows a

multivariate Gaussian distribution

the multivariate distribution can be represented by an ellipsoid
that has a 70% chance of containing the real parameters





- using the L_2 norm constraints parameters to an ellipsoid = least squares

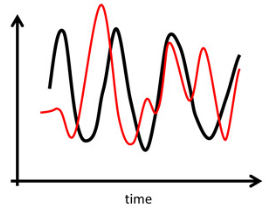
$$|z - z_{obs}|_2 = \sum_i (z_i - z_{obs,i})^2$$

- what about other norms ?

$$|z - z_{obs}|_1 = \sum_i |z_i - z_{obs,i}|$$

$$|z - z_{obs}|_\infty = \max_i |z_i - z_{obs,i}|$$

...



- regularization

$$|z - z_{obs}|_2 + \alpha |P|_1$$

$$|z - z_{obs}|_2 + |CP|_2$$

$$|z - z_{obs}|_2 + |CP|_2 + \beta |P|_1$$

...

Lasso

Bounded Variable Least Squares (BVLS)

Danzig selector (DS)

Elastic Net

high order Tikhonov

Tikhonov

Weighted Lasso

Truncated Singular Value Decomposition (TSVD)

There are many ways to estimate parameters from data !

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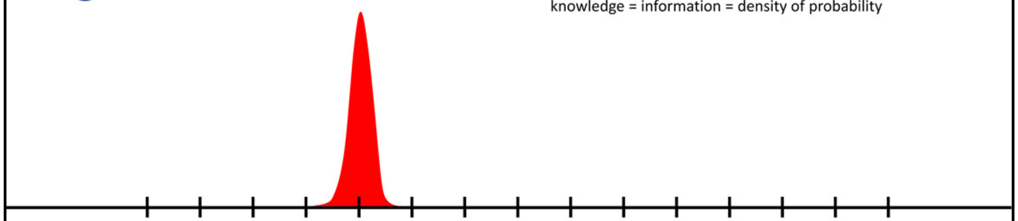
- how is the knowledge of the parameters improved by the data ?

knowledge = information = density of probability



- notation : $f_X(x) = [X]$ probability density of variable X

  $f_{X|Y}(x) = [X \mid Y]$ probability density of variable X, given variable Y
  This is what we are looking for !

- Bayes formula : $[X \mid Y] = \alpha \times [X] \times [Y \mid X]$

- $[P \mid z_{obs}] = [P] \times [z_{obs} \mid P] = [p_1] \times [p_2] \times \ldots \times [z_{obs,1} \mid P] \times [z_{obs,2} \mid P] \times \ldots$

$$= \prod_i [p_i] \times \prod_i [z_{obs,i} \mid P]$$

$$= \prod_i [p_i] \times \prod_i \exp\left(-\frac{(z_{obs,i} - (MP)_i)^2}{2\sigma_\varepsilon^2} \right)$$

$$= \prod_i [p_i] \times \exp\left(\sum_i -\frac{(z_{obs,i} - (MP)_i)^2}{2\sigma_\varepsilon^2} \right)$$

$$= \prod_i [p_i] \times \exp\left(-\frac{\left| z_{obs} - MP \right|^2}{2\sigma_\varepsilon^2} \right)$$

$$= \exp\left(-\frac{\left| z_{obs} - MP \right|^2}{2\sigma_\varepsilon^2} \right)$$

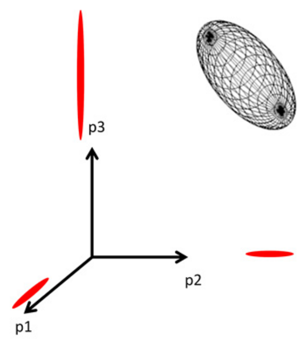
the most probable set of parameters is the one that minimizes $\left| z_{obs} - MP \right|^2$
... the least square solution

... a multivariate Gaussian distribution $\exp\left(-\frac{(P-\mu)^T C^{-1} (P-\mu)}{2} \right)$

➡ the least square solution is the optimal method for extracting information

- if the a priori knowledge on the individual parameters = Gaussian distributions

$$[P\,|\,z_{obs}]=[P]\times[z_{obs}\,|\,P]=\underbrace{[p_1]\times[p_2]\times\ldots}_{\text{multivariate Gaussian distribution}}\times\underbrace{[z_{obs,1}\,|\,P]\times[z_{obs,2}\,|\,P]\times\ldots}_{\text{multivariate Gaussian distribution}}$$



... the updated knowledge = multivariate Gaussian distribution

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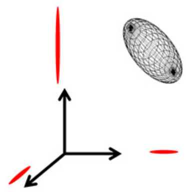
- updated knowledge = multivariate Gaussian distribution



center : $(N^T N)^{-1} N^T Z_{obs}$

parameter 1σ uncertainties : $\sigma_\varepsilon \sqrt{diag\ (N^T N)^{-1}}$

where $N = \begin{pmatrix} M \\ C_P^{-1/2} \end{pmatrix}$ and $Z_{obs} = \begin{pmatrix} z_{obs} \\ C_P^{-1/2} \mu_P \end{pmatrix}$

with prior information, the solution is still given by the least squares

only true for Gaussian priors

- equivalent to Tikhonov regularization :

$$\left|NP - Z_{obs}\right|_2 = \left|MP - z_{obs}\right|_2 + \left|\sigma_\varepsilon C_P^{-1}(P - \mu_P)\right|_2$$

- Any form of regularization can be interpreted as accounting for additional information

very important to know in order to use regularization correctly

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SUMMARY :

- in presence of Gaussian noise
  least squares are the optimal method for extracting information from observations
  - least squares constrain parameters into an ellipsoid
  - any improvement requires additional information
- prior information on parameters can be easily accounted for,
  if prior distributions are Gaussian the solution is then given by Tikhonov regularization
  - Tikhonov regularization also constrains parameters to an ellipsoid
- any form of regularization can be interpreted as accounting for additional information
  this is interpretation necessary in order to apply the regularization correctly

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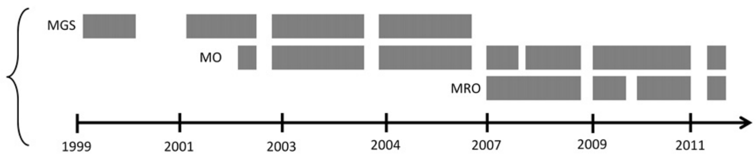
objective

determine and extract asteroid mass information from Mars range data

data

approximately 11000 Mars ranging observations



model ≈ model used to build the DE423 planetary ephemeris

Folkner 2010

adjusted parameters :

a total of about 400 parameters

- **343** asteroid masses
- 12 initial conditions for the orbits of Earth and Mars
- other (solar plasma correction, constant biases)

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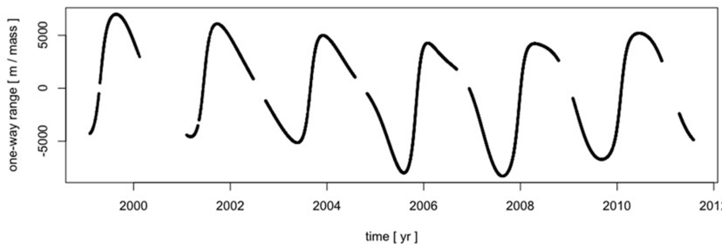
constraints on parameters given by least-squares : $P = (M^T M)^{-1} M^T z_{obs}$ and $\Delta P = \underset{\text{1 m}}{\varepsilon} \sqrt{diag\ (M^T M)^{-1}}$

matrix of partials

obtained by finite differences



fitting the model parameters by simple least squares :

huge uncertainties

in practice, the inversion of the covariance matrix is impossible (the matrix is close to singular)

the range data alone provides no information regarding asteroid masses

more information is needed

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what do we know about the 343 asteroid masses ? ... they are positive

depend on diameters and densities

asteroid diameters

depend on absolute magnitude and albedo (Bowell et al. 1989)

absolute magnitude

$$D_{[\ \mathrm{km}\ ]} = \frac{1329}{\sqrt{\rho}} 10^{-0.2H}$$

albedo

WISE	MIMPS	SIMPS
Masiero et al. 2011	Tedesco et al. 2004	Tedesco et al. 2004
~ 100000 diameters	~ 100 diameters	~ 1000 diameters

300 / 343 diameters are known to 10%

40 / 343 diameters are known to < 35%

only 4 / 343 diameters are undetermined

- classical approach to introduce prior information :
  introduced in Williams 1984

  - splitting asteroids into major and minor objects

    ~ 20 asts
    masses fitted individually

    ~ 320 asts
    diameter
    taxonomy class (C/S/M)
    masses determined assuming constant density in the 3 taxonomy classes
    3 parameters fitted

  - number of parameters reduced
  - involves 2 hypotheses
  - uncertainty estimation not obvious

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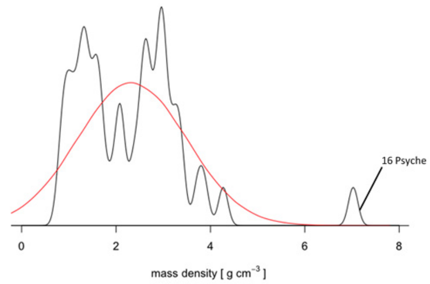
asteroid mass densities



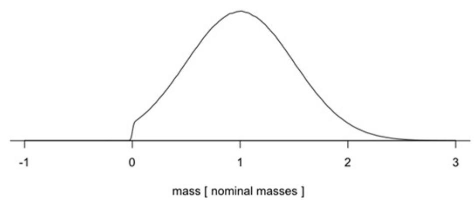
density distribution is approximately Gaussian :

mean ≈ 2.2 g cm^{-3}
deviation ≈ 1 g cm^{-3}

prior information on masses for asteroids with determined diameters :

- nominal mass
  computed from radiometric diameter and mean density

- 1σ uncertainty = 0.5 nominal mass
  the true mass can reasonably be at 0 or at twice the nominal mass



truncated Gaussian distribution

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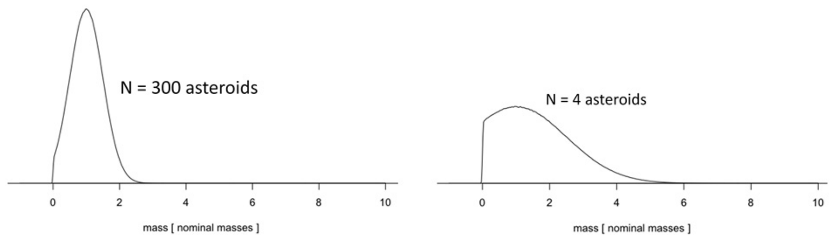
what do we know about the 343 asteroid masses :



- Tikhonov regularization with Gaussian priors

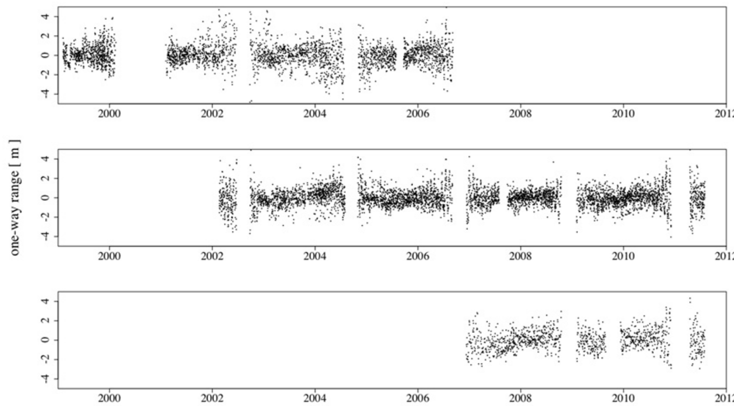
constraints on parameters given by least-squares : $P = (N^T N)^{-1} N^T Z_{obs}$ and $\Delta P = \varepsilon \sqrt{diag\ (N^T N)^{-1}}$

- Tikhonov regularization with truncated Gaussian priors
  solved with NNLS algorithm (Lawson & Hanson 1974)

fitting the model parameters to Mars range data

post-fit residuals :

- MGS
  1σ = 1.1 m
- ODY
  1σ = 0.9 m
- MRO
  1σ = 1.0 m



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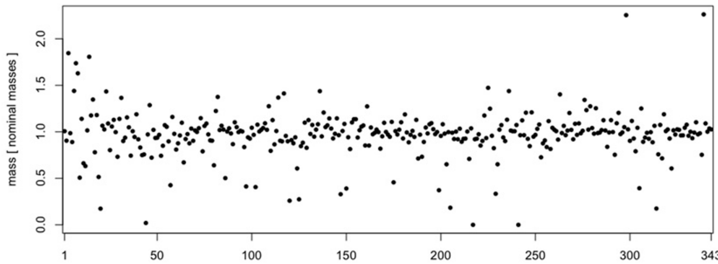
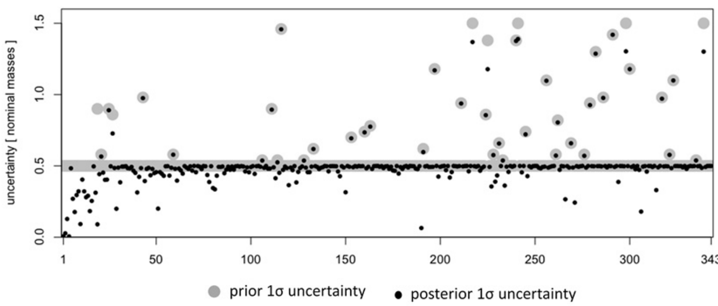
asteroid masses :



uncertainties :

the range data constraints relatively few masses



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30 asteroid masses are determined to better than 35%

of the adjusted mass

		GM [$km^3 s^{-2}$]	uncertainty (%)
4	Vesta	17.13	0.6
1	Ceres	62.53	0.6
2	Pallas	12.42	3.0
3	Juno	1.82	6.9
10	Hygiea	6.85	8.1
324	Bamberga	0.68	8.8
7	Iris	1.07	10.2
704	Interamnia	2.99	14.4
15	Eunomia	1.53	15.6
14	Irene	0.52	16.1
19	Fortuna	0.44	17.5
8	Flora	0.33	18.1
6	Hebe	0.70	18.7
16	Psyche	1.68	18.9
52	Europa	2.37	20.7

		GM [$km^3 s^{-2}$]	uncertainty (%)
532	Herculina	0.79	22.3
18	Melpomene	0.25	26.4
29	Amphitrite	0.59	27.3
88	Thisbe	0.76	27.4
511	Davida	1.94	27.6
13	Egeria	0.78	27.6
23	Thalia	0.13	28.1
31	Euphrosyne	1.83	28.2
89	Julia	0.30	31.4
423	Diotima	0.76	31.8
654	Zelinda	0.19	32.2
164	Eva	0.13	33.3
134	Sophrosyne	0.17	33.4
505	Cava	0.12	33.7
41	Daphne	0.37	34.1

asteroids determined by other methods :

- spacecraft or multiple system
- Mars ranging (classic approach) Konopliv et al. 2011
- close encounters - Baer et al. 2001

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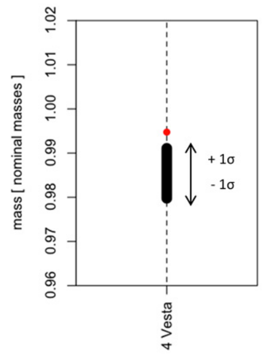
comparison with other mass determination :

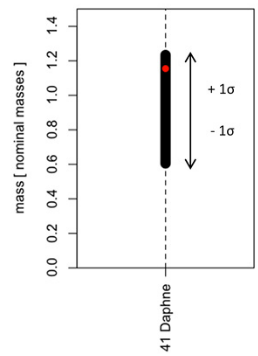
DAWN tracking

A. Konopliv



41 Daphne binary system observation

Conrad et al. 2008



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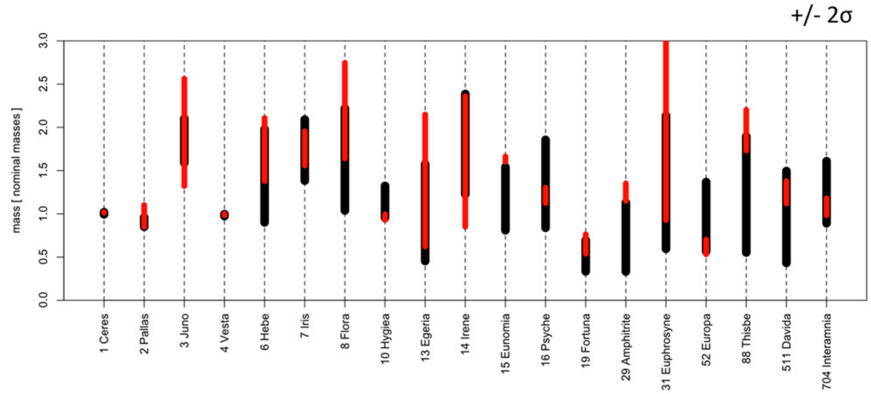
close encounters
Baer et al. 2011

+/- 2σ



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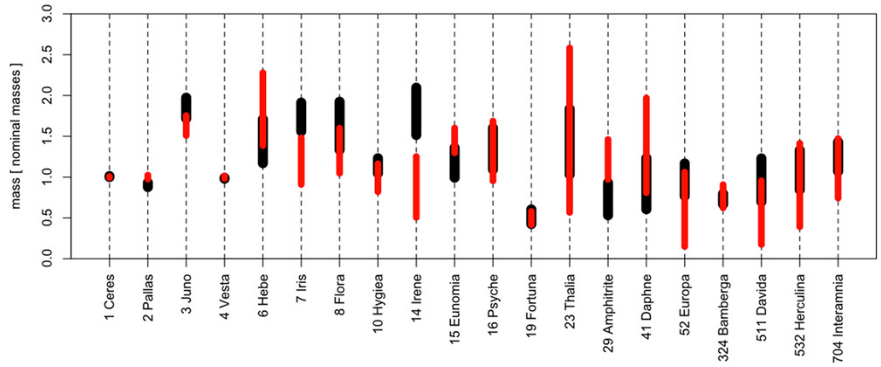
Mars ranging, classic approach

Konopliv et al. 2011

+/- 1σ



our masses compare well with determinations obtained by others

Tikhonov regularization performs at least as well as the classic approach

while relying only on available knowledge of asteroid diameters and densities

(no taxonomy information, no assumption of constant density, no selection of individually adjusted parameters)

CONCLUSION :

Tikhonov regularization appears as a good alternative to the classical approach

- performs well :
  - 30 asteroid masses can be determined from range data to better than 35%
  - compare well with estimates obtained elsewhere
- offers a rigorous framework to treat prior information :
  - avoids choices / hypotheses necessary in the classic approach
  - guarantees that we cannot do better without additional information

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- acknowledgements :
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THANK YOU !